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What proportion of a cohort of women has what proportion of the children? For the cohort of US women born in 1930, the 36 percent who had four or more children accounted for fully 63 percent of the cohort’s issue. Ten percent of the women were childless, and the 32 percent of the women with one or two children accounted for only 17 percent of the progeny. This concentration of reproduction—a prolific fraction of the cohort producing most of the offspring and a sizable fraction of the cohort leaving relatively few descendants—constitutes what might be called a division of labor.

In this research note we analyze this division of labor in the United States, for various cohorts of women who completed their fertility (at age 49) between 1917 and 1980. We first consider the question, What proportion of women have half the children? We then use concentration curves to analyze the more general question, What proportion of women have what proportion of the children?

What proportion of women have half the children?

A serendipitous constellation of three arresting statistics sparked our interest in the concentration of reproduction:

— In a synthetic cohort of women following 1980 US fertility rates, about a quarter of the women would give birth to half of the children.

— A volume tracing the history of the eugenics movement reports that "A 1906 demographic study of a number of London districts . . . substantiated [Karl Pearson’s] warning that half of each succeeding generation was produced by no more than a quarter of its married predecessor . . ." (Kevles, 1985: 74).
— The section on "demography and population genetics" in the *International Encyclopedia of Social Sciences* asks us to "recall a well-known calculation made by Karl Pearson in connection with Denmark. In 1830, 50 percent of the children in that country were born of 25 percent of the parents" (Sutter, 1968: 105).

These three statistics are, it turns out, three points on a complicated pattern and not three instances of a universal demographic constant. That makes the pattern interesting: how does what we have termed the "have-half"—in this context, the proportion of women (or of married women or of parents) who have half the children—change over time, across countries, with the level of fertility? For the present note, we narrow our concern to reporting some of our findings about the changing concentrations of reproduction among US women.

Figure 1 graphs three summary measures of the concentration of reproduction among cohorts of US women born between 1868 and 1931. The curve labeled "Women's have-half" appears fairly stable and fluctuates slightly; on the other hand, the difference between the lowest and highest points on the curve is substantial. The lowest point is reached for the cohort of women born in 1906: 19 percent of these women had half the children. The highest point is reached for the last cohort, the women born in 1931: 27 percent of these women accounted for half the offspring. Thus, the pattern is one of decreasing and then increasing equality in the distribution of offspring.

**FIGURE 1** Three summary measures of the concentration of reproduction among cohorts of US women born 1868–1931
Using concentration curves

To gain a deeper understanding of the significance of the general level and the fluctuations in the women’s have-half measure, it is useful to consider the entire set of “have-statistics” on concentration of reproduction. What proportion of a cohort has a quarter of the children? What proportion of the children do the most prolific quarter of the women account for? All such indexes are summarized in a single curve, the concentration curve, which relates cumulative proportions of women to cumulative proportions of children.  

Figure 2 presents the concentration curve for the cohort of US women born in 1930. It can be seen that 10 percent of the women had 25 percent of the children, that 90 percent of the women had all the children (i.e., 10 percent of the women were childless), and that the top 25 percent of the women had 49 percent of the children. The seven dots on the curve mark different family sizes: the leftmost dot represents women with seven or more children. Thus it can also be seen from the curve that the 21 percent of the women who had five children or more accounted for 44 percent of the children—and, corre-

FIGURE 2  The concentration of reproduction among US women born in 1930
spondingly, that 44 percent of the children came from families of five children or more.

The concentration curve is derived from Heuser’s compilation of statistics on proportions of women in the 1930 cohort at each level of completed childbearing (Heuser, 1976, updated by the National Center for Health Statistics), presented in the first and second columns of Table 1. The second column gives the proportion of women in the cohort who had, respectively, seven or more children, six children, five children, down through no children. The first column gives the average number of children born to women in each of these categories. Thus, the 7 percent of the women who had seven or more children had 8.51 children on average.3

The fourth column presents the product of the numbers in the first and second columns. The sum of the figures in this column gives the average number of children for all women, 3.16. Dividing the numbers in the fourth column by 3.16 gives the proportion of the offspring attributable to women in the different categories, as presented in the fifth column.

Finally, the proportions in the second and fifth columns are cumulated in the third and sixth columns, respectively. Figure 2 was constructed by plotting these pairs of points and then connecting them with a straight line and connecting the end points with the zero/zero and one/one points.

### What proportion of women have what proportion of the children?

We generated concentration curves for each cohort of women born from 1868 through 1931. The boundaries of this set of curves are defined by three curves, given in Figure 3. The 1906 curve, that is, the curve for the cohort of women born in that year, tends to be the outermost curve, whereas the 1931 curve tends to be the innermost. However, the earliest curve, for the 1868 cohort,
The concentration of reproduction among US women born in 1868, 1906, and 1931

cuts across the other two curves, going from innermost to outermost. Thus, for both the 1868 and 1931 curves, the most prolific eighth of the women had about a quarter of the children, and for both the 1868 and 1906 curves, the most prolific half of the women had about five-sixths of the children.

To develop a fuller understanding of the concentration curves in Figure 3, it is useful to trace the changes over time in three summary measures of the concentration of reproduction. This is done by the various curves shown in Figure 1. As noted earlier, the women’s have-half fluctuates from 19 percent (for the cohort of women born in 1906) to 27 percent (for the cohort born in 1931). The proportion of the childless women varies from 21 percent (for the 1906 birth cohort) to 10 percent (for the 1931 cohort). Because the proportion childless tends to decrease when the proportion of women who have half the children increases, we also examined the proportion of mothers who have half of the children. We hypothesized that this mothers’ have-half curve might be more or less flat; it turns out to be relatively smooth (but far from flat), ranging from 24 percent for the 1906 cohort to 30 percent for the 1931 cohort.
Conclusion

Populations are heterogeneous. Averages hide that heterogeneity. Knowing that the cohort of US women who completed their fertility in 1980 had 3.16 children on average tells only part of the story. The 36 percent of these women who had four children or more accounted for 63 percent of the children. Equivalently, 63 percent of the children were born in families with four or more children. Facts like these about the concentration of reproduction substantially add to our understanding.

Demographers often use frequency distributions to study population heterogeneity: for the cohort of women completing their fertility in 1980, 10 percent of the women had no children, 10 percent had one child, 22 percent had two children, and so on. Concentration curves, and the statistics on concentration of reproduction associated with them, are also useful in uncovering, analyzing, summarizing, and comparing the amount and nature of diversity within populations. Frequency distributions and concentration curves complement each other, providing two perspectives that help deepen comprehension. On the one hand, for example, 10 percent of women in the 1930 cohort had a single child; on the other hand, only 3 percent of children were single children. On the one hand, only 7 percent of women had seven children or more; on the other hand, 20 percent of children were born to mothers with seven or more children.

Populations are heterogeneous in many different ways, relating not only to fertility, but also to mortality, morbidity, marriage, mobility, and other demographic phenomena. Analysis of population concentration can shed light on many topics of demographic interest. Furthermore, concentration analyses may be relevant to policy decisions, especially those relating to the targeting of an intervention. If one-fifth of women are bearing half the children, perhaps policies to reduce (or to increase) births should be directed toward this group.

Notes

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1 At IIASA, we initiated analysis of a range of such research questions (Goodwin and Vaupel, 1985a, b, c; Vaupel and Goodwin, 1986). This research is being continued by Lutz (Lutz and Vaupel, 1987).

2 Economists use concentration curves to study inequality in the distribution of income and to study the concentration of business activity. Such curves, proposed by Lorenz (1905), are often called Lorenz curves. Statistics on concentration of reproduction are frequently used in summarizing concentration curves; they are sometimes referred to as fractiles or percentiles. An economist, for instance, might summarize the concentration of
wealth by saying that the top 10 percent of the population has 70 percent of the total wealth. A rule of thumb, the so-called 20–80 (or 80–20) rule, common in economic and business circles, is that 20 percent of the population (of books, files, employees, or whatever) accounts for 80 percent of the usage, output, problems, or whatever (Kenner, 1986). Duncan (1957) discusses the use of concentration curves by demographers, with emphasis on spatial concentration; Duncan also surveys indirectly relevant research by Lotka (1925), Zipf (1949 a and b), and others, on the distribution of population sizes of, e.g., cities. A recent review of segregation and diversity measures in population distributions is given by White (1986). In his captivating analysis of family sizes of children and family sizes of women, Preston (1976) analyzes the relationship between the average number of children per woman and the average number of siblings per child. We review the use of concentration curves and related measures, such as the Gini coefficient and the Simpson index, with a focus on application in the life sciences, in Goodwin and Vaupel (1985 a, b, c).

3 Heuser’s data include birth rates for women with eight or more children, but it is only possible to calculate the total number of children—and hence, the average number of children—born to women with seven or more children.

References


