

## OVER-TENURED UNIVERSITIES: THE MATHEMATICS OF REDUCTION\*

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Most universities anticipate an era of retrenchment over the next decade or two. The heady period of expansion fueled by the baby boom and by the jump in the percentage of high school graduates enrolling in college is now yielding to the lean years of the baby bust. As college enrollments fall, universities face a seller's market in attracting new students; on the other hand, as job openings decline, universities enjoy a buyer's market in hiring new faculty. It is thus understandable that university administrators are striving to maximize their flexibility in determining the composition of their professorial work force—to respond to shifts in student interest, to trim dead wood, and to seize opportunities to hire transcendent new Ph.D.'s.

The employment guarantees of tenure make it harder to achieve this flexibility. Most universities, however, are entering the era of retrenchment with more than three-quarters of the faculty holding tenure. Consequently, many university administrators are pondering ways to reduce the tenure ratio. Attention has been focused on making tenure harder to get, i.e., on reducing the proportion of junior faculty who, after the usual trial period of five or six years, are granted tenure. This article compares this strategy with two alternatives—increasing the attrition of less-worthy tenured faculty and lengthening the average time to tenure.

It turns out that in most cases even a relatively modest decrease in the tenure ratio, say from 80% to 67%, would require a radical reduction in the percentage of new faculty who can be expected to be granted tenure—e.g., from 50% to 25%. Remarkably enough, the same impact on the tenure ratio of such of halving of the chances of tenure can be achieved either by doubling average time to tenure or by doubling the attrition of already tenured faculty.

Since faculty have to leave the university if denied tenure, a drastic cut in the chances of tenure may severely diminish the loyalty and the dedication to teaching of junior faculty as well as making faculty recruitment more difficult. Therefore, universities should seriously weigh alternative strategies. A variety of means exist for encouraging attrition, including holding salaries down, giving bonuses for early retirement, and making promotion from associate to full professor more selective. Lengthening the time to tenure should also be considered: many junior faculty would prefer to face a 50% chance of tenure after ten years rather than a 25% chance after five years; furthermore, the ten-year trial period would enable better evaluation of performance, as well as requiring only half as much recruiting of new junior faculty each year.

The desirability of the alternative strategies depends on the specific attributes and preferences of particular universities. No panacea emerges—and it is by no means clear that a university, given the drawbacks, would be wise to attempt to reduce its tenure ratio. The simple mathematics developed in this article, however, does demonstrate that reducing the chances of tenure is certainly not the only feasible way, and in many cases probably not the best way, of reducing the tenure ratio.

(ORGANIZATIONAL DESIGN; HIGHER EDUCATION; TENURE POLICY)

### 1. Three Policy Variables

The baby bust has left most universities feeling over-tenured. Universities seeking, whether wisely or not, to reduce the ratio of tenured faculty to total faculty without

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substantially altering total faculty size can choose among a variety of policies that alter three key variables—(1) the proportion of untenured faculty granted tenure, (2) the attrition rate of tenured professors, and (3) the average length of time before tenure is granted or denied.

Some simple mathematics clarifies the relationships among these variables:

–Let  $r$  be the ratio of tenured faculty to total faculty.

–Let  $p$  be the proportion of newly-hired untenured faculty who can be expected to eventually become tenured.

–Let  $u$  be the average number of years untenured faculty stay at the university until they either are tenured or leave.

–And let  $a$  be the attrition rate of tenured faculty. This attrition rate multiplied by the size of the tenured faculty gives the number of tenure slots open to junior faculty each year. Thus, the attrition rate measures the proportion of tenured faculty who are not replaced by tenured faculty hired from other universities.

In equilibrium these four variables are interrelated according to a formula which, with apologies to the A.A.U.P., might be called the  $au/p$  rule:

$$r = 1/(1 + au/p). \quad (1)$$

The rule is perhaps not quite so remarkable as it might seem at first glance since the variables  $a$ ,  $u$ , and  $p$  were artfully defined so as to produce a simple formula. The proof of the rule is, as a consequence, elementary. The number of tenured positions opening up each year is given by  $a \cdot T$ , where  $T$  is the total number of tenured positions. Since only the proportion  $p$  of untenured faculty eventually become tenured, in equilibrium  $aT/p$  untenured faculty are hired each year. Since these faculty stay an average of  $u$  years (before either being granted tenure or leaving for whatever reason), the total number of untenured professors is given by:

$$U = (aT/p) \cdot u. \quad (2)$$

The value of  $r$  is, by definition, given by:

$$r = T/(T + U). \quad (3)$$

Substituting equation (2) for  $U$  and then dividing through by  $T$  yields equation (1).

To get a feel for the  $au/p$  rule, consider the case of some hypothetical university, U.S.U. Suppose that at U.S.U. the value of  $a$  is 0.03. That is, 3% of the tenure slots open up each year for junior faculty. This would occur if 4% of the tenured faculty leave each year, but a quarter of them are replaced by tenured faculty from other universities. Since in equilibrium the mean length of stay of tenured faculty is given by the inverse of the proportion who leave per year, a value of 4% implies a mean length of tenure of 25 years. Suppose that  $u$  is 4 years: since some untenured faculty wait 6 or more years before being granted (or denied) tenure, but others are granted tenure or leave the university within a year or two, 4 years may be a plausible average. And suppose that  $p = 0.5$ , so that newly-hired junior faculty members face a fifty/fifty chance of eventual tenure. Then the equation implies that  $r$  equals 81%: just over four-fifths of the faculty will be tenured.

At first glance, this may seem to be a counter-intuitive statistic: if 50% of the junior faculty are given tenure, why is the tenure ratio not 50%? The key to this conundrum is that untenured faculty serve only 4 years, on average, but tenured faculty remain with the university for an average of 25 years. Indeed, with an attrition rate of 3%, it

takes 33 years, on average, for a tenure slot to open up for a junior faculty member. If two are in the running, they contribute 8 years. And 33 divided by 33 plus 8 is 81%.

If U.S.U., say, tripled the attrition rate  $a$ , while holding  $u$  and  $p$  constant, the new value of  $r$  would equal 58%. The same impact could be gained by tripling  $u$  while holding  $a$  and  $p$  constant, by cutting  $p$  by a factor of three while holding  $a$  and  $u$  constant, or by increasing  $a$  and  $u$  and decreasing  $p$  by a factor equal to the cube root of three or, roughly, 1.44. In this sense, then, the variables  $a$ ,  $u$ , and  $p$  have the same impact. The cost, however, of pulling the different policy levels that change  $a$ ,  $u$ , and  $p$  may differ. In particular, for many universities it may be less costly to implement policies that increase the attrition rate by some factor or that increase the average time to tenure by that factor than it would be to cut the probability of tenure by the same factor.

The value of  $r$  does not depend on the current ratio of tenured faculty to total faculty: given the values of  $a$ ,  $u$ , and  $p$  determined by university policy, the tenure ratio approaches  $r$  regardless of the current level of the tenure ratio. The speed of approach is fairly rapid: as shown below, equilibrium in most cases is nearly attained within a decade or two. Consequently, the impact on  $r$  of changes in  $a$ ,  $u$ , and  $p$  is of interest to decisionmakers even if they are more concerned about the next few years than about the long-run equilibrium.

## 2. Making Tenure Harder to Get

Most universities are attempting to reduce  $r$  largely by reducing  $p$ , i.e., by making tenure harder to get. Equation (1) implies that if  $a$  and  $u$  are kept at current levels, then to achieve some new tenure ratio,  $r'$ , the required  $p$  (call it  $p'$ ) is given by:

$$p' = aur' / (1 - r'). \quad (4)$$

The variable  $p$  is the product of two other variables— $s$ , the proportion of newly-hired untenured faculty who will remain at the university until the tenure decision, and  $\pi$ , the proportion of those who remain who are granted tenure. The most obvious strategy for influencing  $p$  is to set a policy that determines  $\pi$ . The lower  $\pi$  is, the lower  $s$  will tend to be, so that a policy that lowers  $\pi$  will have double leverage on  $p$ .

Suppose, for example, that U.S.U. wants to reduce its tenure ratio from 81% to 67%, keeping  $a$  at rate 3% and  $u$  at 4 years. Then equation (4) implies that the proportion of junior faculty granted tenure will have to fall from the current level of 50% to 24%. Further suppose that currently  $\pi$  is 60% and  $s$  is 83% (producing the  $p$  of 50%). To lower  $p$  to 24% it might be sufficient to cut  $\pi$  to 40%—as long as this change induced a decrease in  $s$  to 60%.

The effect of the value of  $\pi$  on the value of  $s$  is likely to differ across academic disciplines, depending on supply and demand. In fields with an excess supply of junior faculty—such as anthropology, history, or literature—the value of  $s$  may be close to 1.0 regardless of the value of  $\pi$ . In such fields, a  $p$  of 0.50 might be the product of a  $\pi$  of 0.52 and an  $s$  of 0.96; reducing  $p$  to 0.24 might thus require sharply cutting  $\pi$  to as low as 0.25. On the other hand, in fields with an excess demand for junior faculty—such as accounting or health policy—the value of  $s$  might be considerably less than 1.0 even for high values of  $\pi$ , as well as being sensitive to changes in the value of  $\pi$ . In these fields, a  $p$  of 0.50 might be the product of a  $\pi$  of 0.80 and an  $s$  of 0.63;  $p$  could be reduced to 0.24 by cutting  $\pi$  to 0.60 if that induced a reduction in  $s$  to 0.40.

These examples imply an important but often ignored fact: a policy that sets a similar  $\pi$  across different academic departments may result in substantially different values of  $p$ , and hence substantially different tenure ratios, across departments. To the extent a university seeks to reduce its tenure ratio by reducing  $\pi$ , it should consider tailoring its policy so that different values of  $\pi$  are set in excess-supply versus excess-demand fields.

In addition to this consideration, it is important not to lose sight of the effect of a tenure policy on  $p$ . In the case of U.S.U., regardless of the value of  $\pi$  and the induced value of  $s$  necessary to attain the required value of  $p$  of 0.24, it is clear that the cut in  $p$  from 0.50 to 0.24 is a radical change that will affect the loyalty and dedication to teaching of junior faculty (even if they have few other employment opportunities), as well as making recruitment more difficult. Cutting  $p$  in half may be least deleterious to the university in fields of excess faculty supply, despite the drastic cut in  $\pi$  required in these fields. The prospect is so unappetizing, however, that universities like U.S.U. should carefully weigh alternative policies that change the values of  $a$  or  $u$ .

### 3. Increasing the Attrition Rate

Equation (1) implies that to achieve a tenure ratio of  $r'$  by changing  $a$ , the new value of the attrition rate must be:

$$a' = (p/u) \cdot (1 - r')/r'. \quad (5)$$

Thus to reduce its tenure ratio to 67% while keeping  $p$  at 50% and  $u$  at four years, U.S.U. would have to double the attrition rate of its tenured faculty from 3% to 6.2%. To understand the alternative policy levers university administrators control for achieving such a change, it is useful to view the attrition rate as a function of five factors:

– $a_1$  and  $a_2$ , the proportion of tenured associate professors and of full professors, respectively, who leave the university each year,

– $f$ , the proportion of tenured faculty who are full professors, and

– $h_1$  and  $h_2$ , the proportion of tenured openings at the associate professor and at the full professor level, respectively, that are filled by individuals hired from outside the university.

Then,

$$a = [a_1 \cdot (1 - f) \cdot (1 - h_1) + a_2 \cdot f \cdot (1 - h_2)]. \quad (6)$$

This formula suggests several strategies for increasing the attrition rate. The values of  $h_1$  and  $h_2$  could be reduced: fewer tenure openings could be filled by outside hiring. Alternatively,  $a_1$  and  $a_2$  could be increased: less worthy tenured faculty could be encouraged to leave the university by holding their salaries down, by assigning them larger or less interesting classes, by obliging them to fulfill their university responsibilities, or by buying them out by, for example, giving bonuses for early retirement. In some cases it may also be possible to lower the average age of retirement by more strictly enforcing retirement rules or by changing the rules—to the extent this is possible under prevailing U.S. laws designed to protect the rights of older people in general and older academics in particular.

Since tenured associate professors tend to be more mobile than full professors and since universities tend to be less obligated to them in recognition of past service, a

selective strategy of increasing the attrition rate of tenured associate professors may be desirable. One way of doing this would be to make promotion to full professor difficult—as difficult, say, as promotion to tenured associate professor. This strategy, which would increase  $a_1$  by lowering  $f$ , would tend to increase the quality of the faculty: it is deleterious to the university—and unfair—to deny qualified junior professors tenure while promoting tenured associate professors whose performance has not met expectations. On the other hand, to the extent that anxiety about promotion chances selectively increased the attrition of the best teachers and researchers among the tenured associate professors—and to the extent tenured associate professors in general are better teachers and researchers than full professors—the costs of this strategy might outweigh its benefits.

An interesting variant on this strategy would *increase* the value of  $h_2$ : a greater proportion of openings at the full professor level would be filled from outside the university. Such a change would encourage tenured associate professors (and, to a lesser extent, junior faculty) to leave the university. To the extent the new value of  $a_1$  exceeded  $a_2$ , the overall attrition rate would increase and hence, *ceteribus paribus*, the tenure ratio would decrease.

The attrition rate of tenured associate professors could also be raised by lengthening the average time before promotion to full professor. This strategy would not only increase  $a_1$  but also would decrease  $f$ : holding faculty size constant, the longer the time to promotion, the lower will be the proportion of tenured faculty who are full professors. Thus, to the extent  $a_1$  exceeds  $a_2$ , this strategy would provide double leverage on  $a$ .

Consider, again, the case of U.S.U. Suppose that  $f$  equals 0.75: some three-quarters of tenured faculty are full professors. Suppose that  $h$  equals 0.25: some 25% of tenure openings are filled by hiring from outside. And suppose that  $a_1$  and  $a_2$  both equal 0.04: one in twenty-five of the tenured associate professors and of the full professors leave the university each year. Then equation (4) implies that  $a$  equals 0.03: some 3% of the tenure slots open up and are filled each year by promoting junior faculty.

To achieve a tenure ratio of 67% without changing  $p$  or  $u$ , it was calculated earlier that  $a$  would have to increase to 6.2%. This increase could be accomplished in numerous ways. For instance, the proportion of tenured faculty who are full professors could be decreased from three-quarters to three-fifths by policies that delayed promotion to full professor and that made such promotion more difficult. In addition, the proportion of tenure openings at the full professor level filled by outside hiring could be reduced from a quarter to a fifth, and the proportion at the associate professor level, from a quarter to a tenth. Finally, the attrition rate of full professors could be increased from 4% to 7% and the attrition rate of tenured associate professors from 4% to 8%. A policy that produced these effects might well prove to be less damaging to U.S.U. than the alternative strategy of cutting tenure chances in half.

Two additional indices may be of interest here. The number of slots at the full professor level opening up each year for tenured associate professors is given by:

$$a_2 \cdot (1 - h_2) \cdot f \cdot T, \quad (7)$$

(where  $T$  is the total size of the tenured faculty). The number of tenured associate professors is simply

$$(1 - f) \cdot T. \quad (8)$$

Therefore in equilibrium,  $\tau$ , the average number of years newly-tenured associate professors can expect to have to wait before being promoted to full professor, is the ratio of (7) and (8), or

$$\tau = (1 - f) / [a_2 \cdot (1 - h_2) \cdot f]. \quad (9)$$

Under current U.S.U. conditions of  $f = 0.75$ ,  $a_2 = 0.04$  and  $h_2 = 0.25$ , the average time is 11.1 years. Under the policy outlined above, the average time would increase by less than a year to 11.9 years.

A second interesting index measures the proportion of newly-tenured associate professors who can be expected to stay in the university and eventually be promoted to full professor. If it is assumed that everyone has to wait exactly  $\tau$  years before being promoted, then this retention rate is given by:

$$(1 - a_1)^\tau. \quad (10)$$

Under current U.S.U. conditions, nearly two-thirds of newly-tenured associate professors will eventually become full professors at U.S.U. Under the option described above, this percentage falls to about two-fifths.

#### 4. Increasing Time to Tenure

The third kind of strategy available to university administrators involves increasing  $u$ , the average number of years untenured faculty stay at the university until they either are granted tenure or leave. Consider then, the effect of changing  $u$ , holding  $a$  and  $p$  constant. It follows from equation (1) that

$$u' = (p/a) \cdot (1 - r') / r'. \quad (11)$$

Thus, for U.S.U. to reach a tenure ratio of 67% while keeping  $p$  at 50% and  $a$  at 3%, the average time to tenure or exit would have to double from 4 years to 8 years.

Essentially there are two ways of increasing  $u$ . First, a greater proportion of untenured faculty could be recruited fresh out of graduate school rather than from other universities. New Ph.D.'s are generally willing to wait longer before being granted or denied tenure than experienced faculty hired from outside. Second, new faculty could be hired (or current contracts renewed) with the understanding that tenure review will be delayed, on average, until, say, the beginning of a new Ph.D.'s tenth year at the university and an experienced faculty member's fifth or sixth year. Many junior faculty members would prefer to face a 50% chance of tenure after ten years rather than a 24% chance after five years. Furthermore, the ten-year trial period would enable better evaluation of an individual's work, as well as requiring only half as much recruiting of new untenured faculty each year.

A careful evaluation of a policy of delaying tenure review should distinguish between  $u$ , the average time to tenure or exit, and  $t$ , the average time to tenure. If a proportion  $s$  of newly-hired untenured faculty remain at the university until the tenure decision, i.e., for an average of  $t$  years, and if the remainder leave the university after an average of  $\lambda$  years, then

$$u = s \cdot t + (1 - s) \cdot \lambda. \quad (12)$$

Since  $t$  exceeds  $\lambda$ ,  $t$  also exceeds  $u$ . A policy that increases  $t$  will also tend to decrease  $s$  and thus will have double leverage on  $u$ .

### 5. Getting There

Since none of the strategies suggested above will yield the desired tenure ratio immediately, it is informative to look at how quickly they reduce the tenure ratio.

Let  $r_0$  be the current tenure ratio. Let  $r'$  be the desired ratio. And let  $r_1$  be the ratio one year after the new tenure policy is instituted. Then a good measure of progress is

$$\rho = (r_0 - r_1)/(r_0 - r'). \quad (13)$$

In U.S.U.'s case,  $r_0 = 81\%$  and  $r' = 67\%$ . If  $r_1 = 78\%$ , then

$$\rho = 0.03/0.14 = 0.21.$$

Thus, 21% of the gap between the current tenure ratio and the desired ratio would be closed in the first year.

The value of  $r_0$  is given by equation (1). The value of  $r_1$  is given by:

$$r_1 = r_0 \cdot (1 - a') + (1 - r_0) \cdot p'/u', \quad (14)$$

where  $a'$ ,  $p'$ , and  $u'$  are the new values of  $a$ ,  $u$ , and  $p$ . This can be shown as follows. Let  $S$  be the total size of the faculty: as noted earlier, throughout this paper  $S$  is assumed to be constant. By definition  $r_1$  is simply  $T_1/S$ , where  $T_1$  is the size of the tenured faculty at time 1. The value of  $T_1$  is given by

$$T_1 = T_0 \cdot (1 - a') + U_0 \cdot p'/u', \quad (15)$$

since the first term of this formula gives the number of tenured faculty retained from the previous year and the second term gives the number of untenured faculty promoted to tenure. Dividing both sides of equation (15) by  $S$ , and then substituting  $r_0 = T_0/S$  and  $1 - r_0 = U_0/S$  yields equation (14).

Some straightforward calculations based on equation (14) yield the elegant result that

$$\rho = a'/(1 - r'). \quad (16)$$

To see this, substitute equation (14) in equation (13) and simplify, getting:

$$\rho = [r_0 a' - (1 - r_0) \cdot p'/u']/(r_0 - r'). \quad (17)$$

Now solve

$$r' = 1/(1 + a'u'/p'), \quad (18)$$

a variant of equation (1), for  $p'/u'$ , as follows:

$$p'/u' = a'r'/(1 - r'). \quad (19)$$

Substituting this result in equation (17), and then simplifying, yields equation (16).

To examine rates of progress in reducing the tenure ratio under different policy options, it is useful to define  $\rho_a$  as the rate of progress produced by changing  $a$  enough to yield  $r'$  and to define  $\rho_u$  and  $\rho_p$  similarly. Then equation (16) implies that

$$\rho_u = \rho_p \quad (20)$$

and that

$$\rho_a = (a'/a) \cdot \rho_p \quad (21)$$

Therefore a policy that doubled  $a$  (from, say, 3% to 6%) would approach the desired tenure ratio twice as quickly as corresponding policies that either decreased  $p$  or increased  $u$ .

The fact that the current tenure ratio nowhere appears in equation (16) implies that the rate of progress  $\rho$  will be constant over time: every year the gap between the existing tenure ratio and the targeted ratio will be closed at the rate of  $\rho$ . Consequently, changing  $a$  will produce the fastest approach to the desired tenure ratio and changing  $p$  or  $u$  the slowest approach.

In the case of U.S.U.,  $\rho_a$  turns out to be 19%, and  $\rho_p$  and  $\rho_u$  to be 9%. Thus, increasing the attrition rate of tenured faculty (from 3% to 6%) will close up the gap between the current tenure ratio and the desired ratio by nearly a fifth every year. On the other hand, either cutting tenure chances (from 50% to 24%) or doubling time to tenure or exit (from 4 years to 8 years) will close the gap by less than a tenth.

Since the gap between  $r_0$  and  $r'$  will be closed exponentially at the rate  $1 - \rho$ , the value of tenure ratio  $y$  years after new values of  $a$ ,  $u$ , and/or  $p$  are chosen is given by:

$$r_y = r' + (1 - \rho)^y \cdot (r_0 - r'). \quad (22)$$

This equation implies that at the progress rate of 19% per year produced by the change in  $a$ , U.S.U.'s current tenure ratio of 81% would fall to 72% in five years and to 69% in nine years. On the other hand, at the progress rate of 9% per year produced by the necessary change in  $p$  or  $u$ , it would have taken eleven years for the tenure ratio to fall to 72% and twenty-one years for it to fall to 69%.

## 6. Getting There Faster

Instead of choosing some  $a$ ,  $u$ , and  $p$  that will gradually achieve and maintain the desired tenure ratio, university administrators may wish to get to the desired ratio more quickly by choosing more extreme values of  $a$ ,  $u$ , and  $p$ . Once the desired ratio is reached, the values of  $a$ ,  $u$ , and  $p$  can then be relaxed to maintain the ratio. Such a desire for speedy reduction in the tenure ratio will enhance the desirability of policies that rely heavily on increasing the attrition rate of tenured faculty, since it is by increasing  $a$  that the quickest reduction in the tenure ratio can be achieved.

Values of  $a$ ,  $u$ , and  $p$  required to reach some tenure ratio  $r_y$  in  $y$  years can be determined iteratively, using a pocket calculator, on the basis of equation (22). Suppose, for instance, that U.S.U. wanted to reduce its tenure ratio of 81% to 67% in 3 years. Leaving  $u$  and  $p$  unchanged, U.S.U. could do this by increasing  $a$  from 3% to 10%. There are, however, no values of  $p$  and  $u$  that could achieve this goal: even if  $p$  equals zero and  $u$  equals infinity, in 3 years the tenure ratio will only fall to 74%.

## 7. Induced Effects

The purpose of the analysis presented so far was to gain insight into how best to reduce the tenure ratio by exploring the effects of pure strategies that change only one of  $a$ ,  $u$ , or  $p$ . If compensating action is not taken, however, a policy that changes one of these variables may induce changes in the other two, reinforcing or weakening the direct effect of the policy.

The easiest case is a change in the attrition rate  $a$ —such a change is unlikely to have much net impact on  $u$  or  $p$ .



Similarly, a change in  $p$  or  $u$  will, in most cases, result in little immediate change in  $a$ . Changes in  $p$  or  $u$ , however, may gradually affect  $a$  in the long run by altering the character of the set of faculty granted tenure. The harder it is to get tenure, the more attractive the tenured faculty will appear to other institutions. This competition may be at least partially offset by some psychological factors increasing faculty loyalty. The longer it takes to be granted tenure, the older the tenured faculty will be. Older faculty tend to be less mobile, but they are also closer to death or retirement. Thus it is unclear whether or not changing  $p$  or  $u$  will significantly affect  $a$ , even in the long run. Some empirical research is needed here.

The most immediate and dramatic induced effects involve the impact of  $p$  on  $u$  and of  $u$  on  $p$ . As the chances of tenure decrease, more junior faculty are likely to leave the university after only two or three years of service. This induced effect will partially offset the direct effect of reducing  $p$  and thus diminish the effectiveness of policies that lower the probability of tenure. On the other hand, increasing  $u$  is likely to decrease  $p$ : as the time to tenure increases, more junior faculty will tend to leave before being reviewed for tenure. This induced effect will reinforce the direct effect of increasing  $u$ , strengthening the efficiency of policies that lengthen the time to tenure.

### 8. Another Perspective

The  $au/p$  rule given in equation (1) implies that the ratio of  $U$ , the number of untenured faculty, to  $T$ , the number of tenured faculty, is given by:

$$U/T = au/p. \quad (23)$$

Substituting equation (12) for  $u$  and  $s \cdot \pi$  for  $p$  yields:

$$U/T = mat/\pi, \quad (24)$$

where the multiplier  $m$  is given by:

$$m = 1 + (\lambda/t)(1/s - 1). \quad (25)$$

Equation (24) provides a somewhat different perspective on tenure policy. The perspective is useful because  $t$ , the average time to tenure, and  $\pi$ , the proportion of those who apply for tenure who are granted tenure, are policy variables more directly under university control than  $u$ , the average time to tenure or exit, and  $p$ , the proportion of newly-hired untenured faculty who eventually become tenured. The equation clearly indicates the correspondance between the effects of changing  $a$ ,  $t$ , or  $\pi$ : for example, doubling the attrition rate will, *ceteris paribus*, have the same impact on the tenure ratio as doubling average time to tenure or as halving the proportion granted tenure.

As before, induced effects may muddy this clear picture. Fortunately,  $t$  and  $\pi$  can, in nearly all cases, be considered as independent policy levers under direct university control. Furthermore, in the short run  $a$  is likely to be independent of  $t$  and  $\pi$  and, for reasons explained above, may be more or less independent even in the long run.

That leaves  $m$ , as given by equation (25). The ratio of  $\lambda$ , the average length of time spent at the university by untenured professors who leave the university before applying for tenure, to  $t$ , the average time to tenure, has to fall between 0 and 1 and may often lie between 0.5 and 0.8. The value of  $s$ , the proportion of untenured faculty who remain at the university until the tenure decision, is likely to be in the range of 0.5

to 0.9. Furthermore, when  $\lambda/t$  is high,  $s$  is also likely to be high, and *visa versa*. Consequently,  $m$  probably usually falls in the fairly narrow range from 1.1 to 1.3. It seems plausible then to assume that even large changes in  $a$ ,  $t$ , or  $\pi$  will induce only small changes in  $m$ . As a first approximation,  $m$  may therefore be treated as a constant that can be ignored in determining the relative impact of changes in  $a$  versus  $t$  versus  $\pi$ .

To the extent there are induced changes in  $m$ , increasing either  $a$  or  $t$  may decrease  $m$  and thus somewhat offset the intended impact. Decreasing  $\pi$ , on the other hand, seems likely to increase  $m$  and thus produce a stronger effect than the first-order approximation would indicate. The approximation, however, is probably roughly right: similar changes in either the attrition rate, the time to tenure, or the probability of tenure will produce about the same long-run effect in the tenure ratio.

### Conclusion

Most universities are attempting to reduce their tenure ratio largely by making it harder to get tenure. To achieve a significant reduction in the tenure ratio, however, chances of tenure have to be cut radically. Although some reduction in tenure chances may be advantageous, alternative strategies that increase the attrition of less-worthy tenured faculty or that lengthen the average time to tenure or exit could achieve the same tenure ratio as a drastic reduction in tenure chances at what would probably be less damage to the quality of the university. An increase in average time to tenure might be more desirable, both for university administrators and for junior faculty, than a substantial cut in tenure chances. An increase in the attrition of tenured faculty would produce a target tenure ratio much more rapidly than a lessening of tenure chances and, indeed, may be the only way of reducing the tenure ratio significantly in a period of under five years. None of these conclusions imply that universities *should* reduce the tenure ratios: given, however, that this is a desired goal, the simple mathematics developed in this paper strongly suggest that universities should seriously consider alternatives to drastic reductions in tenure chances.

The difficulties of implementing such alternative strategies cannot be ignored: increasing the attrition of tenured faculty may require Draconian measures and increasing the probationary period before tenure is granted is a major institutional change not to be taken lightly. Depressing tenure chances is, however, also a radical change and perhaps the most damaging one. Universities should weigh the alternatives carefully, including the alternative of leaving the tenure ratio be.

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